3.17.1. Semantic Problems:
Tautology, Contradiction, and Logical Equivalence

A. **Translate** each English sentence into the formal language and build a **truth table** for that formal sentence. On the basis of the truth table, find a **simpler formal sentence** that's **logically equivalent** to the original; then **translate** that simpler sentence back **into English**.

1. Neko will either eat fish or eat fish.

2. Rex isn't a hippy, and he's also not a hippy.

3. Jack is a cat who either eats flies or doesn't.

4. Either Jack is a cat who eats flies, or he's one who doesn't.

5. Ace is the winner, and he'll receive a trophy unless he doesn't.

6. We're either having truffles, or we're having both truffles and grog.

7. Rex went out, and he did so without taking his umbrella.

8. Jack is a cat, and one who doesn't eat turkey.

9. Gasoline fights are either dangerous or pleasant, and they're also either dangerous or unpleasant.

10. Either Rex finished his homework, or he went to the party without finishing it.
(*Note: the simpler sentence doesn't appear earlier in the truth table.*)

11. Suki cooked breakfast, but she didn't do so without making a mess.
(*Hint*: see the discussion of negated "without" sentences in *3.10. Scope*
Note: the simpler sentence doesn't appear earlier in the truth table.)

B. For each pair of sentences, **translate** both into formal language and build **truth tables** for them to show that the two sentences are **logically equivalent**.

1a. Neither Trixie nor Elvis failed to attend the poker tournament.
1b. Both Trixie and Elvis attended the poker tournament.

2a. We’re having ice cream, and either cake or pie.
2b. Either we’re having ice cream and cake, or ice cream and pie.

3a. Either we’re having ice cream, or we’re having cake and pie.
3b. We’re having either ice cream or cake, and either ice cream or pie.

4a. We’re having ice cream, but neither cake nor pie.
4b. We’re either having ice cream without cake, or ice cream without pie.

5a. Neko is a cat who likes fish, and one who also likes cream cheese.
5b. Neko is a cat who likes both fish and cream cheese.

C. For each numbered group of sentences below, use truth tables to show that all the sentences in that group are **logically equivalent**.

Distribution

1a. $(P \wedge (Q \vee R)$
1b. $((P \wedge Q) \vee (P \wedge R))$

2a. $(P \vee (Q \wedge R))$
2b. $((P \vee Q) \wedge (P \vee R))$

Idempotence

3a. $(P \vee P)$
3b. $(P \wedge P)$
3c. P

Absorption

4a. $(P \vee (P \wedge Q))$
4b. $(P \wedge (P \vee Q))$
4c. P

D. **Translate** each of the following sentences into formal language; then use a **truth table** to decide whether that sentence is a **tautology**, a **contradiction**, or **neither**.

1. Buck is a dog who isn't a dog.
2. Jack is a cat who eats flies.
3. Jack is a cat who either eats flies or doesn't.
4. Jake ordered an Old Fashioned unless he didn't .
5. Unless we're having both ice cream and cake, we're not having ice cream.
6. Unless we're having both ice cream and cake, we're either not having ice cream or not having cake.
7. Unless we're having ice cream, we're not having *both* ice cream *and* cake.
8. Unless we're not having ice cream, we're having either ice cream or cake.
9. Rex passed Chemistry without studying for the Chemistry exam, though he did study for the Chemistry exam.
10. Unless his handwriting is illegible, Dr. Slim's handwriting is both legible and beautiful.
11. Unless his handwriting is illegible, Dr. Slim's handwriting is either legible or beautiful.
12. Dr. Slim's handwriting is both legible and beautiful, even though it's illegible.
13. Nick and Nora both ordered a martini, unless neither of them did.
14. Either Nick or Nora ordered a martini, unless neither of them did.

E. Build a truth table for each of the following sentences, to show that the sentence is a **tautology**.

1. $\sim(P \wedge \sim P)$

2. $(P \vee \sim P)$

3. $(\sim Q \vee (\sim P \vee Q))$

4. $(P \vee (\sim P \vee Q))$

5. $(P \vee \sim(P \wedge P))$

6. $((P \vee Q) \vee \sim(P \wedge Q))$

7. $((P \vee Q) \vee (\sim P \wedge \sim Q))$

8. $(((P \vee Q) \wedge (\sim P \vee Q)) \vee \sim Q)$

9. $(((P \wedge Q) \vee (\sim P \wedge Q)) \vee \sim Q)$

F. **Consistent and Inconsistent Sentences**. Every inconsistent set looked at so far has contained either (i) a single contradictory sentence, such as $\{(P \wedge \sim P)\}$; or (ii) a pair of sentences where one is the negation of the other, such as $\{P, \sim P\}$. Show that neither feature (i) nor feature (ii) is a requirement for being an inconsistent set, by building an inconsistent set which lacks both.